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T.Y.B.Sc. (NCF-NEP) (Sem.-V) Examination March - 2026

Mathematics Major – I : Group Theory – I (Paper MH MJ – 501)

Time: 1 Hour] **[Total Marks: 25**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory. (3) Figures to the right indicate marks of the questions. (4) Follow usual notations.	Student's Signature

Q.1. Answer the following as directed: (Any FIVE) **05**

1. If $(a \cdot b)^2 = a^2 \cdot b^2$; $\forall\ a, b$ in a group G, then prove that G is abelian.
2. Give an example of an abelian group; which is not cyclic.
3. Prove that the relation of *"Congruence modulo H* ($\equiv$ *mod H*)" on G is symmetric; where H is a subgroup of a group G.
4. Let H be a subgroup of a cyclic group G; where $o(G) = 21$ and $o(H) = 7$, then find $i_G(H)$.
5. Define Normal Subgroup. Give an illustration of a normal subgroup of a non-abelian group.

Q.2. Attempt any TWO : **10**

a. Define Abelian Group. Prove that:
   i. Every element a in a group G has the unique inverse in G.
   ii. In a group G ; $(a^{-1})^{-1} = a$; $\forall\ a \in G$.

b. Prove that $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \middle| \, a \in \mathbb{R} / \{0\} \right\}$ is a group under matrix multiplication.

c. Prove that a non – empty subset H of a group G is a subgroup of G if and only if
   i. $a, b \in H \Rightarrow a \cdot b \in H$;
   ii. $a \in H \Rightarrow a^{-1} \in H$.

Q.3 Attempt any TWO: **10**

a. State Lagrange's Theorem. Prove that:
   i. If G is a finite group and a in G, then $a^{o(G)} = e$.
   ii. If p is a prime number and a is an integer such that $p \nmid a$, then prove that $a^p \equiv a (\text{mod } p)$.

b. Let H and K be subgroups of a group G. If $HK = KH$, then prove that HK is a subgroup of G.

c. Let N be subgroup of a group G. Then prove that N is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
